

Partial F-tests and Lack-of-Fit Tests

PARTIAL F-TEST.

Consider full and reduced models

```
> load("Auto.rda")
> attach(Auto)

> reg_full = lm(mpg ~ year + acceleration + horsepower + weight)
```

How to test significance of year and acceleration?

```
> reg_reduced = lm(mpg ~ horsepower + weight)

> anova( reg_full, reg_reduced )
```

Analysis of Variance Table

```
Model 1: mpg ~ year + acceleration + horsepower + weight
Model 2: mpg ~ horsepower + weight
```

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	387	4558.0				
2	389	6993.8	-2	-2435.8	103.41	< 2.2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

The p-value comparing these two models is very significant, so the two variables make a significant contribution for the prediction of mpg, in addition of weight and horsepower.

LACK-OF-FIT.

Here we test linearity by comparing the linear model (reduced) with the model with dummy variables, one for each value of X (full model that does not assume linearity).

```
> reg_reduced = lm(mpg ~ cylinders)
> reg_full = lm(mpg ~ as.factor(cylinders))
```

```
> anova( reg_full, reg_reduced )
```

Analysis of Variance Table

```
Model 1: mpg ~ as.factor(cylinders)
Model 2: mpg ~ cylinders
```

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	387	8544.5				
2	390	9415.9	-3	-871.42	13.156	3.383e-08 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Low p-value shows that the relation of mpg to the number of cylinders is non-linear.

Continuous case – what to do if the X-variable has no repeated values?

```
> round(horsepower/10)*10
[1] 130 160 150 150 140 200 220 220 220 190 170 160 150 220 100 100 100
80
[19] 90 50 90 90 100 110 90 220 200 210 190 90 90 100 100 100 100
90
      < truncated >

> reg_reduced = lm(mpg ~ horsepower)
> hp_rounded = round(horsepower/10)*10
> reg_full = lm( mpg ~ as.factor(hp_rounded) )

> anova( reg_full, reg_reduced )
Analysis of Variance Table

Model 1: mpg ~ as.factor(hp_rounded)
Model 2: mpg ~ horsepower
  Res.Df    RSS  Df Sum of Sq    F    Pr(>F)
1     373 7101.9
2     390 9385.9 -17      -2284 7.0565 6.662e-15 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The full model is significantly better. So, mpg is a non-linear function of horsepower.